Mathematical Modelling of Ground Response Analysis during Earthquake: A Case Study

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Abstract

In this investigation, main objective is to determine the aftershock effect of earthquake. Spectral acceleration analysis has been done. A proper case study has been done using the Coulomb software. It is observed that the peak acceleration is different in three different cases of faulting. Horizontal displacement, vertical displacement, stress and strain analysis and Coulomb stress change has been analysed. Recently earthquake affected areas has been taken into consideration.

Introduction

Earthquake happen when vibrations are caused by the movement of rock along a fault, a fracture that exist in Earth's crust. As the tectonic plates push against, pull away from, grid past, or dive under one another, fault zones are created. Sometimes tensions builds up along a fault and further movement can cause release of energy in the form of seismic waves, or vibrations in Earth's crust. Those vibrations ripple violently through the crust, causing an earthquake. A severe earthquake can produce underground movements – forward and back, up and down, side to side – and wavelike ripples.

Scientist measuring earthquakes mostly used the Richter scale, developed by U.S. seismologists Charles F. Richter and Beno Gutenberg in the 1930s and 1940s. In their logarithmic scale measures only magnitude. Other scales categorize earthquake by other criteria. The moment magnitude scale is based on the seismic moment. The area of rock displaced, the rigidity of that rock and the average distance of displacement.

The Mercalli intensity scale (named for Giuseppe Mercalli, the Italian scientist who originated it) uses Roman numerals to rate an earthquake by its effects on surroundings. California's San Andreas Fault, for example is a zone where the slow sideways movement of slabs has pushed rock formation some 350 miles from their sources. It is very important to understand the aftershock distribution and this can be solved by dictating main shock mechanism. Baer (1) has shown that the major aftershocks and slip along Gulf parallel normal faulting are associated with positioned Coulomb failure stress changes induced by the main shock. The diverse crustal structure which characterizes the Dead Sea transform region has been investigated by many authors, introducing several methods such as gravimetric mapping, seismic profiling and other studies. The density and velocity 2D model were constructed based on seismic study by Gulf (Ben – Azahan et al (2)). Weber et al (3) has given more detailed discussion on the transform structure. Earthquake damages occurred to buildings located at soil sites are more compared to buildings at rock sites, as reported in literature [4-9].

The ground motion analysis is felt essential for the analysis of important structures. Here our focus is to investigate the aftershock effect of earthquake in Nepal (2015) and for the purpose we have used coulomb software.

Coulomb failure stress change

Coulomb failure stress change (ΔCFS) calculation is performed based on Coulomb failure criterion. This criterion has been used to characterize the conditions under which failure occur in rocks. It is required that shear and normal stress on incipient fault plane satisfy conditions analogous to those of friction on pre-existing surface. There are two equations which present the calculations of shear and normal stress (King et al 1994)

$$\sigma_n = \frac{1}{2}(\sigma_1 + \sigma_3) - \frac{1}{2}(\sigma_1 - \sigma_3)\cos 2\beta \tag{1}$$

$$\sigma_{sh} = \frac{1}{2}(\sigma_1 - \sigma_3)\sin 2\beta \tag{2}$$

σ_{sh} is the shear stress on failure plane. σ_n is the normal stress, $\sigma_{1,3}$ is the principal stress and failure plane is oriented at β to the σ_1 axis. To calculate ΔCFS the value of difference between initial σ_1 and final σ_f stress take place.

The normal $\Delta\sigma_n$ and shear $\Delta\sigma_{sh}$ stress change calculations are:

$$\Delta\sigma_n = \sigma_{n,f} - \sigma_{n,i} \tag{3}$$

$$\Delta\sigma_{sh} = \sigma_{sh,f} - \sigma_{sh,i} \tag{4}$$

In the Coulomb criterion, failure occurs on a plane the Coulomb stress change exceeds positive value.

$$\Delta\sigma_f = \Delta\sigma_{sh} - \mu'\Delta\sigma_n \tag{5}$$

Where μ' is the effective coefficient of friction. Localization of the coulomb failure zone is interpreted because it dictates the earthquake trigger. The ΔCFS caused by the main shock rupture efficiently explain the aftershock distribution of the earthquake but there are a lot of factors that can affect these changes.

Equations of motion

The equilibrium equations used for the purpose

Force balance equation:

$$\frac{\partial \sigma_{ji}}{\partial x_j} + F_i = 0 \tag{6}$$

Where $\sigma_{ij} = \sigma_{ji}$ the Cauchy is stress tensor and F_i is body force.

Strain displacement equation is

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{7}$$

Where $\varepsilon_{ij} = \varepsilon_{ji}$ is the strain force and u_i is the displacement. In this case the displacement are prescribed everywhere in the boundary. In this approach the stress and strain are eliminated from formulation leaving the displacement as the unknown to be solved for in the governing equations. First the strain –displacement equations are substituted into the constitutive equations eliminating the strain as unknown

Hooks law equations:

$$\sigma_{ij} = \lambda \delta_{ij} \varepsilon_{kk} + 2\mu \varepsilon_{ij} \tag{8}$$

$$= \lambda \delta_{ij} \frac{\partial u_k}{\partial x_k} + \mu \left(\frac{\partial u_i}{\partial x_i} + \frac{\partial u_j}{\partial x_j} \right) \tag{9}$$

Differentiation yield,

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \lambda \frac{\partial}{\partial x_i} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \tag{10}$$

Substituting the equation yield,

$$\mu \frac{\partial}{\partial x_j} \left(\frac{\partial u_i}{\partial x_j} \right) + (\mu + \lambda) \frac{\partial}{\partial x_j} \left(\frac{\partial u_j}{\partial x_i} \right) + F_i = 0 \tag{11}$$

Where μ and λ are Lame parameters. The governing equations obtained in this manner are called Navier- Cauchy equations.

Aim of Study

The complexity of the crust in the study area can generally affect the ΔCFS distribution. Therefore calculation in which crustal structure is not taken into account may be inaccurate and even misleading. In the current study we present a simulation of the ΔCFS distribution induced by Nepal event using homogeneity and heterogeneity modelling. At the final step we constructed the crustal structure model of the study area.

Fig.1 : Nepal affected area with magnitude and depth

Methodology:

For calculating the aftershock effect on the fault area we have used coulomb software, in which first step is the grid formation and fault location, may by one, two or so many depending on study area. After that we set the proper set of parameters for the least error in the calculation. Then Matlab inbuilt program is used for the complete analysis.

Input parameters and their dimensions:

1. Poisson's ratio: [Dimensionless, -1 to 0.5]; 0.25 is typically used.
2. Young's modulus: 8x 10^5 bars is typically used.
3. Friction coefficient: 0.4 is often used.

Directions, angle and dip:[degrees; dip must be positive]

Grid and Fault positions(x,y) :[km]

Depth: [km] downward is positive

Displacement:[m]

Faults: Right lateral is positive, and reverse slip is positive [m]

Dikes : Opening displacement is positive [m]

Point source : Inflation is positive [m^3]

Regional stress tensor: S1, S2, S3: positive in compression

Output parameters and their dimensions

Displacement: [m] North, East, and Up are positive.

Shear strain:[Dimensionless] Right-lateral is positive

Principal Strain :[dimensionless] Extension is positive (tensor notation)

Dilatational Strain: [dimensionless] Dilatation is positive

Stress : Right lateral and unclamping are positive.

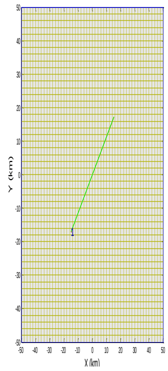


Fig. 2: Fault shown in mesh grid

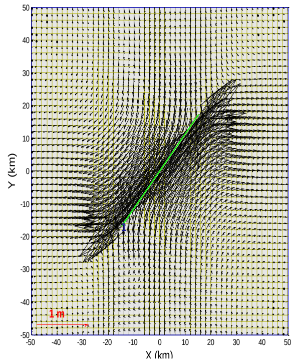


Fig. 3: Horizontal displacement of vectors

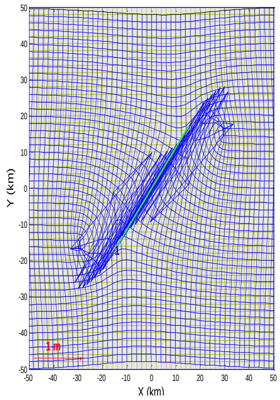


Fig. 4: Horizontal displacement wireframe

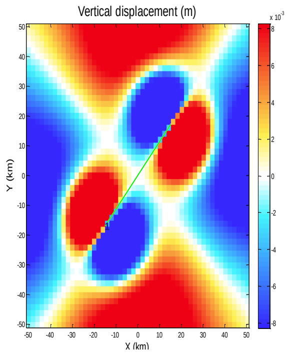


Fig. 5: Vertical displacement distance

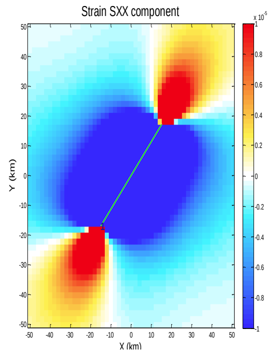


Fig. 6: Strain component (EXX) against

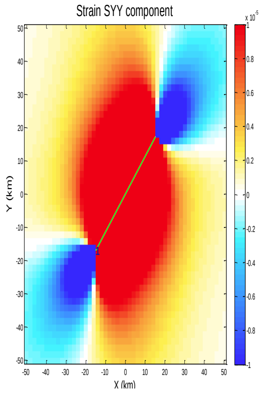


Fig. 7: Strain component (EYY) against distance

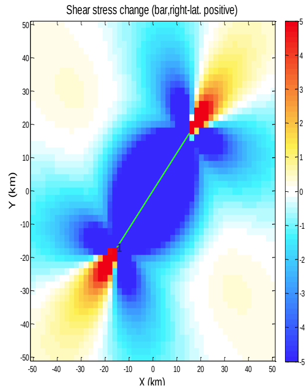


Fig. 8: Shear stress change around fault

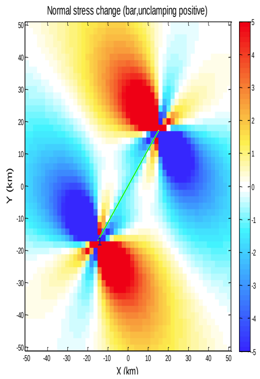


Fig.9: Normal Stress change around fault

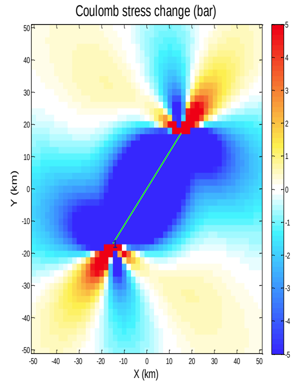


Fig. 10: Coulomb stress change due to fault

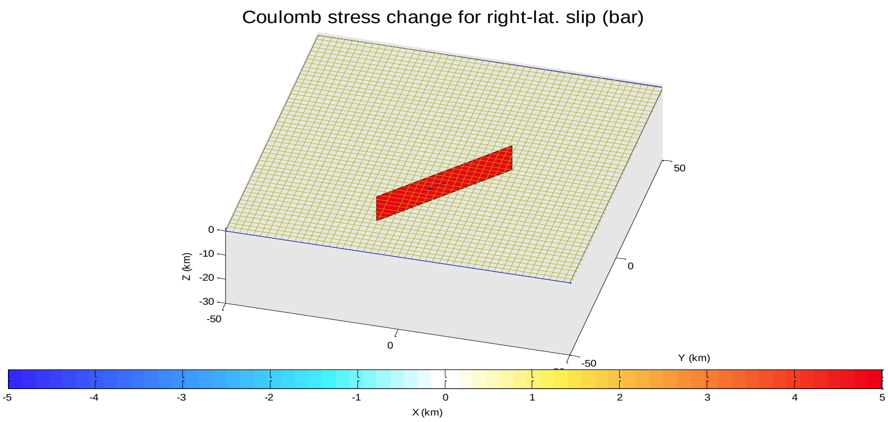
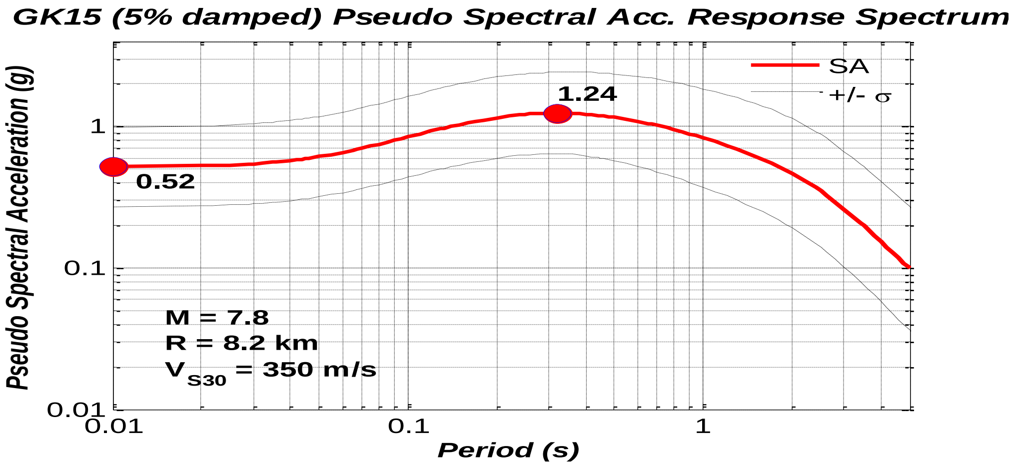


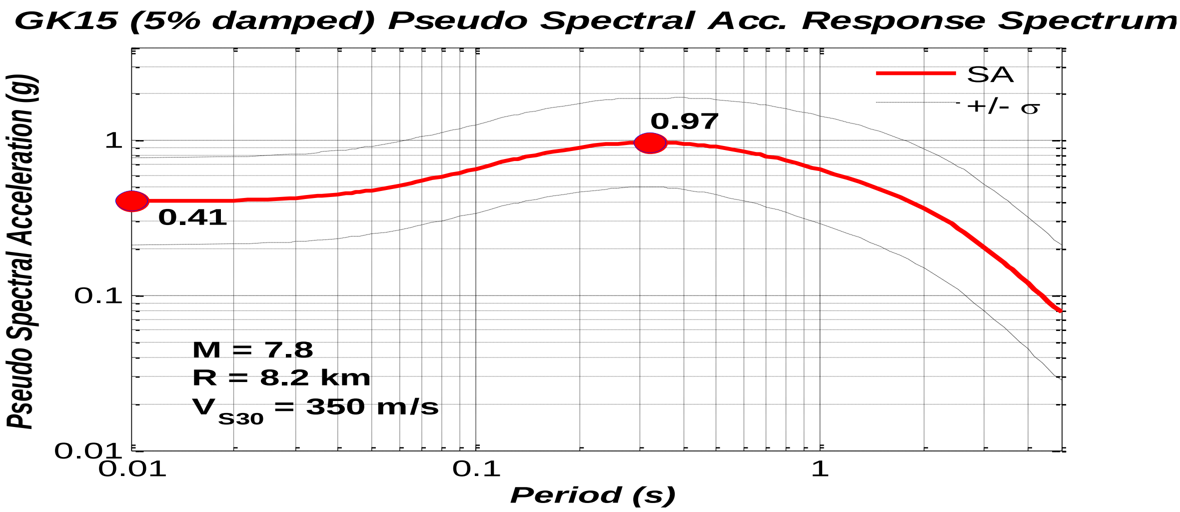
Fig. 11: Coulomb stress change in 3D

Spectral acceleration analysis:

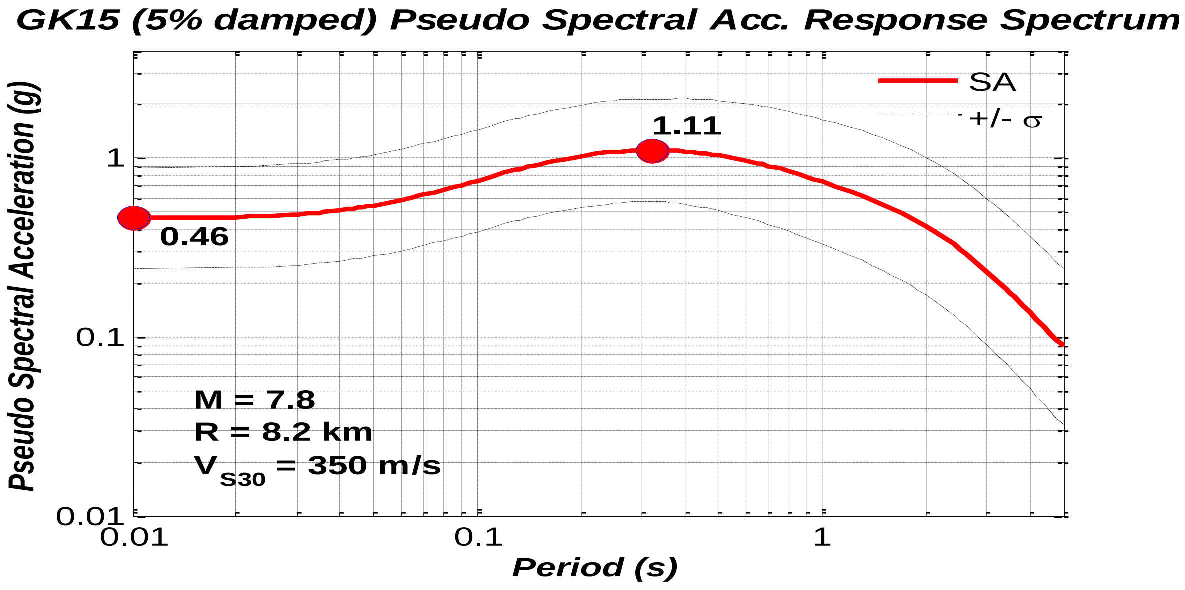
Let M be the moment magnitude and R is the closest distance to fault rupture, VS30 is the average shear wave velocity, is the style of faulting, F = 1.0 for strike slip and normal faulting, F = 1.28 for reverse faulting and F = 1.44 for combination of strike slip and reverse faulting. We have plotted the graphs of three cases by taking the case of moment magnitude of Nepal event. Proper regional quality factor is taken.



This graph is plotted of reverse faulting. It shows smooth increase and then decrease in the graph. It achieves the value 1.24 between period 0.1 to 1. When we deal with strike faulting, we get the graph below



The peak value is 0.97 in this case. The combination of reverse slip and faulting, we get slight change in the graph of spectral acceleration.



Results and discussions

In this investigation, ground motion analysis has been done with the help of coulomb software. A proper fault is taken into consideration in study area with the proper set of input parameters. Half space parameters (Poisson's ratio (0.25) and Young modulus (80000)) are taken and the results are plotted for horizontal displacement, vertical displacement, stress and strain changes. Spectral analysis with different types of faulting has been done with proper moment magnitude and depth.
In Fig. 3 and Fig. 4 horizontal displacement of vectors and horizontal displacement of wireframe has been shown. In Fig. 5 vertical displacement has been shown. In Fig. 6 and Fig. 7 Strain component (EXX) and Strain component (EYY) has been tabulated and plotted.
The variation in stress has been shown in figures. Shear stress and normal stress change has been shown in Fig.8 and Fig.9. The figure values variate according to the colorbar shown in figure. In Fig. 10 Coulomb stress change has been shown.

Conclusions

Seismic risk analysis for a structure require assessment of both the rate of occurrence of future earthquake ground motions (hazard) and the effect of these ground motions on the structure (response). These two pieces are often linked using an intensity measure such as spectral acceleration. The problem for the analysis of ground motion aftershock has been investigated here. For the purpose spectral acceleration has been calculated and plotted in the figure. Structural engineers also utilize spectral acceleration as a basis for analysis of structural response. To view the aftershock effect the Coulomb software has been used. The results critically show the influence of earthquake on the considered study area.

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